Analytical description of stripping foil extraction from isochronous cyclotrons

Stanko S. Tomić*
Department of Physics, University of Surrey, Guildford, Surrey GU2 7XH, United Kingdom

Eugene V. Samsonov†
Laboratory of Nuclear Problems, Joint Institute for Nuclear Research, 141980 Dubna, Russia
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Analytical expressions are derived describing beam parameters on a stripping foil (SF) as a function of radial amplitude of betatron oscillations and of energy gain. The results computed from these expressions are in good agreement with those from numerical calculations. These results indicate the existence of a parametric relationship between radial emittance and energy spread, via the amplitude of radial betatron oscillations. This relationship enables one to generate the working diagram of expected beam parameters on a SF. Such a diagram can be particularly useful for designers of extraction systems, since it gives the relationship between quantities used as extraction system input parameters. The derived analytical expressions can make the design of cyclotrons easier and significantly reduce the need of numerical simulations.

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I. INTRODUCTION

The modern concepts of compact isochronous cyclotrons rely on decreasing the gap between poles of the magnet, thus enabling a better efficiency of the magnetic circuit. This imposes restrictions on the layout of the accelerating structure, as well as the necessity of placing the dees and extraction system components in valleys of the magnet. Such an approach requires a detailed study of the ion beam dynamics in the acceleration and extraction regions. Particularly good matching between accelerated beam emittances and extraction system acceptances is required to provide a high level of extraction efficiency. Change of the ion charge by using a stripping foil (SF) is often employed for beam extraction from the compact cyclotrons. For a successful design of extraction system information is required on the energy spread and emittances of the beam falling on the stripping foil. These beam parameters on the SF are defining parameters of the extracted beam, and, therefore, determine the ultimate performance of the cyclotron.

In this paper analytical expressions are derived to describe beam parameters on the SF in terms of radial amplitude of betatron oscillations and of energy gain. These expressions indicate the existence of a parametric relationship between radial emittance and energy spread, via the amplitude of radial betatron oscillations. This relationship makes it possible to generate the working diagram of expected beam parameters on a SF. The results calculated from the analytical expressions show very good agreement with those from numerical computations. Therefore, the presented analytical expressions can be used rather then performing time consuming numerical simulations, which significantly simplifies the design of cyclotrons.

A comparison of the analytical estimates and numerical results for the beam parameters is carried out for the multiparticle isochronous cyclotron VINCY [1–3]. Extraction of H^-, D^-, $^2H^+$, $^3H^+$, and $^{20}Ne^{5+}$ ions from this cyclotron by means of the SF [4] is proposed. Influence of some parameters, such as the ion type, energy gain per turn, and the radial betatron amplitude, on the beam shape on a SF is discussed and the constraints on beam parameters necessary to meet the requirements of the extraction system are determined. These constraints may be adhered to by careful adjustment of the central region geometry, as well as by appropriate shimming of the magnetic field [5].

II. ANALYTICAL DESCRIPTION OF BEAM PARAMETERS ON A STRIPPING FOIL

A. Radial emittance

A particle in an isochronous cyclotron reaches the radius of a SF by two processes: energy gain per turn and precession of its orbit center. The radial step of the orbit, originating from the energy gain per turn, is given by [6]

$$\Delta r_e = r\,\frac{\gamma}{1+\gamma}\,\frac{\Delta E}{W\nu_r^2}, \tag{1}$$

where r is the radius, $\gamma = 1 + W/E_{amu}$, ΔE the energy gain per turn for one nucleon, W the kinetic energy per nucleon, E_{amu} the atomic mass unit ($=931.49$ MeV), and ν_r the radial betatron frequency. When the kinetic energy of particles is significantly less than the rest mass ($\gamma \approx 1$ and $\nu_r \sim \gamma$), the above equation may be rewritten as

$$\Delta r_e \approx r\,\frac{\Delta E}{2W}\,\frac{1}{(1+W/E_{amu})^2}. \tag{2}$$

The radial orbital step due to the precession motion depends on the amplitude of the radial betatron oscillations A_r, the phase of the radial betatron oscillations ϕ_r, and their frequency ν_r, according to

$$\begin{aligned}\Delta r_p &= A_r[\sin(2\pi\nu_r+\phi_r)-\sin(\phi_r)]\\ &= 2A_r\sin(\pi\nu_r)\cos(\pi\nu_r+\phi_r).\end{aligned} \tag{3}$$

*Email address: s.tomic@surrey.ac.uk
†Email address: sams@nusun.jinr.dubna.su

In order to attain the maximum positive value of Δr_p a particle should satisfy the phase condition $\phi_r=\pi(1-\nu_r)$, which gives

$$\Delta r_p=-2A_r\sin(\pi\nu_r). \tag{4}$$

Since for the compact isochronous cyclotron the tune is typically close to unity ($\nu_r \gtrsim 1$),

$$\Delta r_p\approx 2\pi A_r(\nu_r-1)\approx 2\pi A_r\frac{W}{E_{amu}}. \tag{5}$$

Consequently, the particle with the maximal values of radial amplitude and energy gain will achieve the maximal radial jump ΔR on a SF,

$$\Delta R=\Delta r_e+\Delta r_p\approx r\frac{\Delta E}{2W\nu_r^2}+2\pi A_r(\nu_r-1). \tag{6}$$

As the full radial angular spread of the beam is $\Delta\psi=\Delta P_r/r$, and assuming the distribution of the bunch on the radial phase plane is close to rectangular, the value of the radial emittance ϵ_r can be estimated by

$$\pi\epsilon_r=\Delta R\Delta\psi=\frac{\Delta R\Delta P_r}{r}, \tag{7}$$

where r is the current radius of the beam and $r=R_{\mathrm{SF}}$ at the radius of SF. Therefore, it remains to determine the spread of radial momentum ΔP_r at the SF.

The spread of radial momentum is determined by particles having maximal radial amplitude A_r. The radius of these particles at any azimuth position φ can be written as $r=r_0+A_r\sin(\nu_r\varphi+\phi_r)$, where r_0 is the radius of equilibrium orbit. Therefore, by definition, we have $P_r=dr/d\varphi=A_r\nu_r\cos(\nu_r\varphi+\phi_r)$. Since the particles inside the bunch are randomly distributed, the phase of radial betatron oscillations is in the range of $0<\phi_r<2\pi$ at the azimuth of SF. Then, two particles may always be found with $P_r=A_r\nu_r$ and $P_r=-A_r\nu_r$. For a bunch of particles directed towards a SF, the spread of the radial momentum depends on the dominant mechanism in the particular process of motion. If the radial step due to the energy gain is significantly larger than the one due to precession ($\Delta r_e\gg\Delta r_p$), then the full spread of radial momentum at the SF does not differ from that of the internal circulating beam, and its value is defined by the following expression:

$$\Delta P_r=2A_r\nu_r\approx 2A_r(1+W/E_{amu}). \tag{8}$$

However, if the radial step due to precession motion is much larger than that due to the energy gain ($\Delta r_e\ll\Delta r_p$), the resulting spread of radial momentum is smaller by a factor of approximately 2. In this case several turns are needed for extraction of one bunch of the beam. Having in mind that $\nu_r\gtrsim 1$, in each turn particles with positive value of radial momentum are mostly directed towards the SF. Hence, the spread of radial momentum under these conditions becomes

$$\Delta P_r=A_r\nu_r\approx A_r(1+W/E_{amu}). \tag{9}$$

When Δr_p is comparable to Δr_e one should use an intermediate value of

$$\Delta P_r=\lambda A_r\nu_r\approx\lambda A_r(1+W/E_{amu}), \tag{10}$$

where the parameter $\lambda\in(1,2)$ quantifies the influence that Δr_p and Δr_e have on the radial orbit step.

Substituting now the expressions for the radial spread of particles [Eq. (6)] and for the spread of radial momentum [Eq. (10)] into an expression for the radial emittance [Eq. (7)], we get

$$\epsilon_r=\frac{\lambda^2}{4}A_r\nu_r\left[\frac{\Delta E}{\pi W\nu_r^2}+\frac{4A_r(\nu_r-1)}{r}\right], \tag{11}$$

where the maximum value of the energy gain per turn for one nucleon is given by

$$\Delta E=4\eta U_{\mathrm{rf}}\sin\left(\frac{h\Delta\alpha}{2}\right) \tag{12}$$

and where the factor 4 represents the number of accelerating gaps, $\eta=Z/A$ is the specific ion charge, Z the ion charge, A the ion atomic mass, U_{rf} the amplitude of dee voltage, h the harmonic number, and $\Delta\alpha$ the angular span of the dees. Finally, using Eq. (12), the expression for the radial emittances, Eq. (11), can be rewritten as

$$\epsilon_r=\lambda^2A_r\nu_r\left[\frac{\eta U_{\mathrm{rf}}}{\pi W\nu_r^2}\sin\left(\frac{h\Delta\alpha}{2}\right)+\frac{A_r(\nu_r-1)}{r}\right]. \tag{13}$$

When $\Delta r_e\ll\Delta r_p$ (valid for light ions such as H^-), the precessional motion makes the dominant contribution to the process that determines the shape of the phase space distribution. As a result, the radial phase portrait on a SF takes a triangular form and, as noted above, the spread of radial momentum decreases twofold. Thus, for H^- the resulting emittance on the SF should be evaluated by setting $\lambda=1$. Assuming an approximately rectangular form of the beam in the radial phase plane (an assumption valid when $\Delta r_e\gg\Delta r_p$, which is satisfied for heavy ions such as $^{20}Ne^{5+}$, i.e., in cases of large turn separation), we can estimate the value of radial emittance ϵ_r by setting $\lambda=2$. In the cases of ions having intermediate masses (D^-, $^2H^+$, $^3H^+$), the corresponding values of the scaling parameter λ can be found from the results of numerical computations, as shown below.

B. Energy spread

To estimate the energy spread, a bunch of particles (Fig. 1) is considered at the instant when the outer edge of the bunch has just touched the SF. The bunch is bent because particles located at the edges have a lower energy gain per turn than those in the center. This particular moment (which we will take to be $t=0$) corresponds to the minimal energy of the extracted beam. The resulting energy spread depends on the number of turns k for which all particles of the bunch hit the SF. In the case of single turn extraction ($k=1$), the energy spread of the extracted beam is equal to that of the internal circulating beam, as determined by the bunch phase

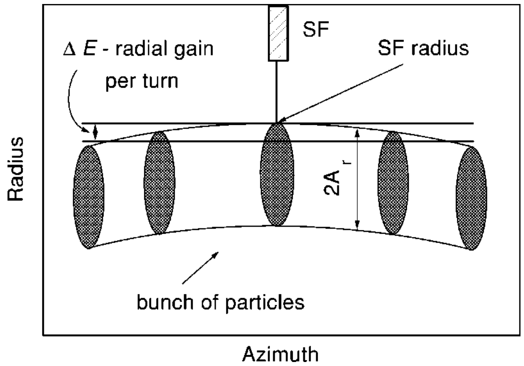

FIG. 1. Schematic view of the bunch of particles in the vicinity of the stripping foil.

width. Since the VINCY cyclotron works in a regime of multiturn extraction by the SF, only this phenomenon is considered here. For such multiturn extraction by a SF, the maximum value of energy is determined by particles that reach the SF after several turns of acceleration. During the first turn of extraction, particles with a maximum value of A_r fall on the SF. Then, in each subsequent turn, another part of the bunch, containing particles with maximum amplitudes at a given moment, will be extracted. This is due to the dependence of particle phase on the betatron oscillations, as seen on the azimuth of the SF. This process will continue, over a number of turns equal to $A_r/\Delta r_e$, for any azimuth cross section of the bunch, leading to an energy spread of $(A_r/\Delta r_e)\Delta E$. Since a bunch of particles is curved in the real space as shown in Fig. 1, an additional energy spread approximately equal to ΔE, is provided by the first turn of the extraction process. The minimal extracted energy corresponds to the particle azimuthally shifted on the left and right from the center of the bunch. Thus, the resulting energy spread is equal to

$$\Delta W=\left(\frac{A_r}{\Delta r_e}+1\right)\Delta E. \tag{14}$$

By substituting Eq. (2) and dividing by $2W$ we finally obtain the expression for the relative energy spread

$$\frac{\Delta W}{W}=\pm\left(\frac{A_r\nu_r^2}{r}+\frac{\Delta E}{2W}\right). \tag{15}$$

It should be noted that, in the case of multiturn extraction by a SF, the relative energy spread is a linear function of the radial amplitude and has a weak dependence on the energy gain per turn.

III. RESULTS AND DISCUSSION

A. Initial particle conditions

For each type of beam considered, the position of the accelerated equilibrium orbit (AEO) [7] was found beforehand at a radius of approximately 10 cm. To determine the position of the AEO with high accuracy we applied an iterative procedure. In this procedure, we stochastically generated a few hundred particles in the radial phase space around the static equilibrium orbit (SEO), which had been determined previously using the CYCLOPS code [8]. Particles were then accelerated for several turns (in this case 20), during the course of which the particle with the minimum amplitude of radial betatron oscillations was selected. The initial conditions of this particle (r_i, p_{ri}) were then taken as the position of the new equilibrium orbit. A new set of particles is generated around this one and is accelerated for several turns, again finding the particle with minimum radial oscillations. When damping of the radial oscillations disappears the iteration procedure is terminated. It should be noted that the required number of iterations does not depend on the phase space density as strongly as it does on the value of the radial emittance area overlapping the AEO. For all ions, the amplitude of radial betatron oscillations of the reference particle is minimized below 0.12 mm, i.e., the AEO is obtained with the same accuracy.

A set of particles is then randomly generated using a Gaussian distribution inside the five-dimensional (r,p_r,z,p_z,ψ) phase volume (initial energy spread is neglected) located around the position of the AEO at the starting azimuth θ_0 (45° upstream from the axis of symmetry of the first dee). For all types of ions, the number of particles in each computation is set to 1000. To model the acceleration process the equations of motion are integrated. The simulation of the particle energy gain is performed for four acceleration gaps and an analytical description is used for the electric field [9]. Both coherent and incoherent initial radial amplitudes ranging from 1 to 6 mm and a bunch phase width $\Delta\psi$ taken to span 15° to 40° rf are employed, according to ion type (Table I). It is important to note that the AEO is chosen for the center of the bunch. Therefore, during the acceleration process the amplitudes of the radial oscillations

TABLE I. Beam parameters on stripping foil. Positions of AEOs are at an angle $\theta_0=45°$ anticlockwise from the axis of symmetry of the first degree.

Ion type	Initial energy (MeV/n)	Position of AEO r (cm)	P_r (cm)	Energy gain per turn (keV/n)	Final energy (MeV/n)	rf bunch phase (deg)	Harmonic number
H^-	0.726	9.287	0.126	135	61	15	1
$^2H^+$	0.255	7.926	0.287	125	29.5	40	2
$^3H^+$	0.170	9.562	0.350	90	13	40	3
$^{20}Ne^{5+}$	0.126	10.932	0.428	80	7.8	40	4

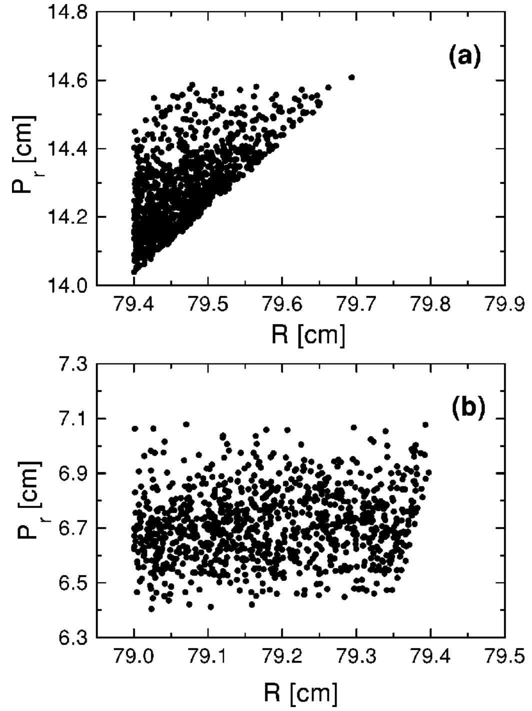

FIG. 2. Radial emittances on the stripping foil for H^- (a) and $^{20}Ne^{5+}$ (b) ions.

of those particles located at the edges of the bunch, and having the same initial position in the radial phase space as the central particles, become larger than the radial oscillations of those located in the center.

The regime of cyclotron operation considered here is that of the maximum attainable energy of ions in the cyclotron VINCY [10]. Table I gives some of the parameters used in the present computations. Acceleration of each particle is continued until the particle radius becomes larger than the radius of the SF boundary at definite azimuth positions. These SF positions are determined [4] during the development of the VINCY extraction system.

B. Radial emittances

In Fig. 2, the numerically calculated particle positions for H^- and $^{20}Ne^{5+}$ ions are shown on the SF radial phase plane. These results are obtained using the fact that the maximum probability of radial amplitudes inside the bunch just before the SF is $\sim$4–5 mm. No magnetic field imperfections ($B_1 = 0$) are used in this calculation. The extraction system is positioned at a range of radii $R_{SF} \in [70,86]$ cm and angles $\theta \in [225°,260°]$. It is obvious that the phase portrait transforms from triangular to rectangular if the ion mass increases. Moreover, the radial step of the orbit due to the energy gain increases and the effect of precessional motion is smaller for the heavier ions. The maximum value of ΔR estimated from Eq. (6) agrees well with the numerical results. The same is observed when comparing analytical and numerical values of ΔP_r for H^- and $^{20}Ne^{5+}$.

Figure 3 reveals an almost linear dependence of radial emittances at the stripping foil on the radial amplitudes existing inside the circulating beam. This superlinear dependence is explained by the fact that, in all cases, the precessional contribution to ΔR is much smaller than that of energy gain. To fit numerical values of emittances and their corresponding analytical estimates using Eq. (13), the scaling parameter λ is chosen to be 1.18 and 1.52 for $^2H^+$ and $^3H^+$, respectively.

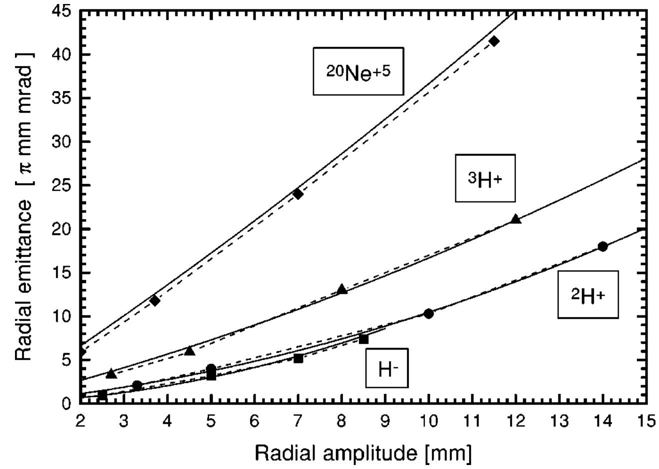

FIG. 3. Radial emittances on a stripping foil vs amplitude of the radial oscillations inside the circulating beam. Solid lines, analytical estimates; points and dashed lines, numerical calculations for four types of ions.

C. Energy spread

Figure 4 shows the distribution of particle energy on the SF as a function of particle phase (divided by the harmonic

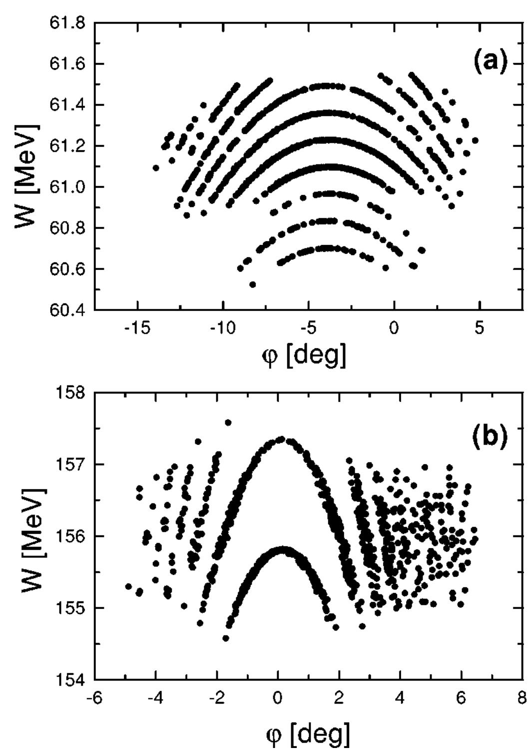

FIG. 4. Distribution of particle energy on the SF for (a) H^- and (b) $^{20}Ne^{5+}$ ions vs their rf phase.

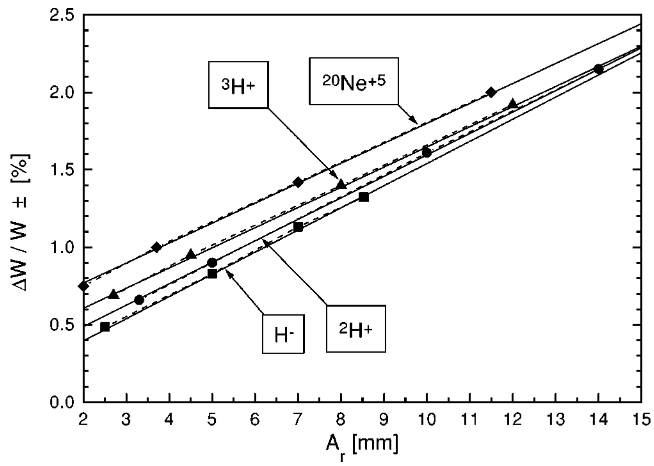

FIG. 5. Relative energy spread for four types of ions as a function of radial amplitude: analytical estimates (solid lines) and results of numerical calculations (points and dashed lines).

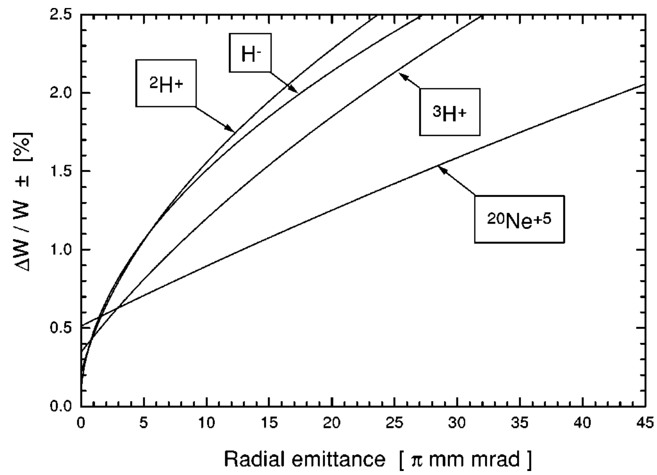

FIG. 6. Energy spread of the beam on a stripping foil vs its radial emittance for four types of ions.

number) relative to the moment of maximum energy gain. From this, one can clearly see the numbers of turns required for extraction, particularly in the center of the bunches. This value varies from 7 in the case of H^- to 2 in the case of $^{20}Ne^{5+}$. These graphs also display the additional energy spread arising during the first turn of extraction due to the bunch phase width.

A comparison of the energy spread obtained from Eq. (14) with the numerically computed value is shown in Fig. 5 as a function of radial amplitude for four types of ions. Most particles, at the instant of falling on the SF, have radial betatron oscillations with amplitudes in the range 4–5 mm. Using histograms we have determined this value for all ions and for all acceleration conditions. A good agreement can be noticed between the two sets of results. Furthermore, the relative energy spread is a linear function of radial amplitude and depends only slightly on ion type. For example, in order to achieve $\Delta W/W < \pm 1\%$ on the SF, the maximum radial amplitude should not exceed 7 mm in the case of H^- and 5 mm in the case of other ions.

D. Working diagram

The results obtained show a parametric relationship between the energy spread and the radial emittance of the beam thrown on a SF. The amplitude of the radial oscillations in the circulating beam takes the role of a parameter. One can plot a working diagram in the space $(\epsilon_r, \Delta W/W)$, representing the main results of the computations. Figure 6 shows this diagram generated for the VINCY cyclotron. To trace the beam accurately inside the extraction system it is essential to consider the related quantities of both emittance and energy spread, for any given ion type. As an example, this diagram shows that for all ion types, if the emittance is smaller than 5π mm mrad, the energy spread does not exceed $\pm 1.1\%$. However, if the emittance is 15π mm mrad, for all ions except $^{20}Ne^{5+}$, the corresponding energy spread is not less than $\pm 1.5\%$. Only in the case of $^{20}Ne^{5+}$ is the energy spread still around $\pm 1\%$.

IV. CONCLUSIONS

A simple analytical description is given of the process that occurs when particles are thrown on a stripping foil. It turns out that the main parameters of interest, such as radial emittance and energy spread, depend linearly on the amplitude of the radial oscillations inside the circulating beam. This conclusion is confirmed by the results of numerical calculations for different types of ions and allows one to establish their relationship by using the amplitude of radial betatron oscillations as a parameter. A working diagram for the particle distribution in the emittance/energy spread parameter space can be easily created and used by researchers involved in the development of extraction systems. On the other hand, constraints on the beam parameters, as determined by the properties of the extraction systems, can be simply transformed into requirements for the radial amplitude of betatron oscillations. These requirements should be taken into account by the designer of the central region in the cyclotron, since the parameters of radial motion in a cyclotron are determined mostly by the central region geometry and by the magnetic field imperfections in this region.

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